NAG Toolbox for MATLAB

s13ac

1 Purpose

s13ac returns the value of the cosine integral

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos u - 1}{u}\,du, \qquad x > 0$$

via the function name, where γ denotes Euler's constant.

2 Syntax

```
[result, ifail] = s13ac(x)
```

3 Description

s13ac calculates an approximate value for $\mathrm{Ci}(x)$.

For $0 < x \leq 16$ it is based on the Chebyshev expansion

$$\mathrm{Ci}(x) = \ln x + {\sum_{r=0}}' a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < x < x_{\mathrm{hi}}$,

$$\mathrm{Ci}(x) = \frac{f(x)\sin x}{x} - \frac{g(x)\cos x}{x^2}$$

where $f(x) = {\sum_{r=0}}' f_r T_r(t)$ and $g(x) = {\sum_{r=0}}' g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x) = 0$ to within the accuracy possible (see Section 7).

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: **x** > 0.0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The function has been called with an argument less than or equal to zero for which the function is not defined. The result returned is zero.

7 Accuracy

If E and ϵ are the absolute and relative errors in the result and δ is the relative error in the argument then in principle these are related by

$$|E| \simeq |\delta \cos x| \text{and } |\epsilon| \simeq \left|\frac{\delta \cos x}{\mathrm{Ci}(x)}\right|.$$

That is accuracy will be limited by ***machine precision*** near the origin and near the zeros of $\cos x$, but near the zeros of $\mathrm{Ci}(x)$ only absolute accuracy can be maintained.

The behaviour of this amplification is shown in Figure 1.

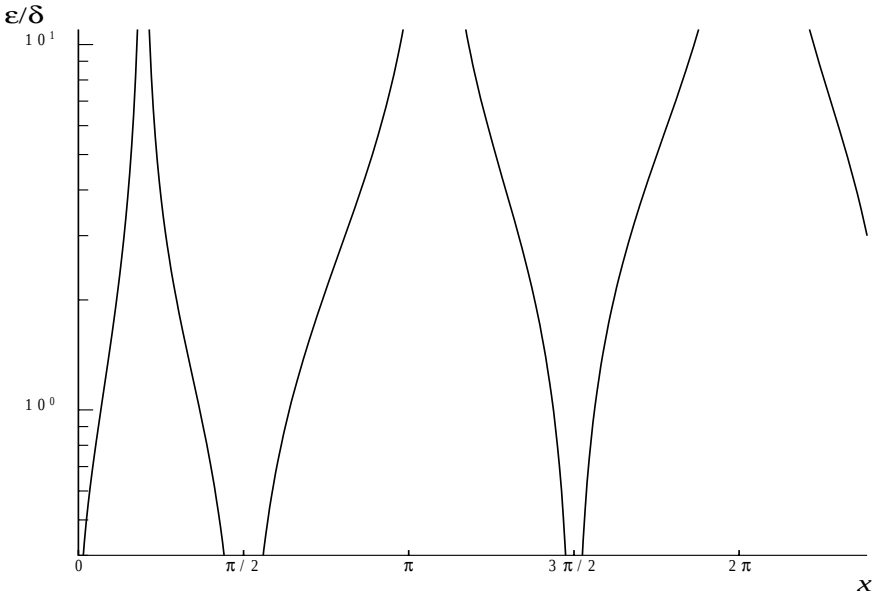

Figure 1

For large values of x, $\mathrm{Ci}(x) \sim \dfrac{\sin x}{x}$ therefore $\epsilon \sim \delta x \cot x$ and since δ is limited by the finite precision of the machine it becomes impossible to return results which have any relative accuracy. That is, when $x \geq 1/\delta$ we have that $|\mathrm{Ci}(x)| \leq 1/x \sim E$ and hence is not significantly different from zero.

Hence x_{hi} is chosen such that for values of $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x)$ in principle would have values less than the ***machine precision*** and so is essentially zero.

8 Further Comments

None.

9 Example

```
x = 0.2;
[result, ifail] = s13ac(x)
```

```
result =
   -1.0422
ifail =
           0
```